Suppression of Chemical Turbulence Using Feedbacks and Forcing

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We report the suppression of spatiotemporal chaos observed in a spatially extended chemical system. In one spatial dimension, under appropriate parameter conditions the model system exhibits transition to turbulence via backfiring of pulses. Suppression is achieved using different feedback and forcing techniques, some of which are applicable in actual experimental situations. Results from the application of some of these strategies to a single (uncoupled) oscillator (two-dimensional O.D.E (ordinary differential equations) system) are presented to demonstrate similarities in the dynamical response of a single system and an extended system under the influence of external feedbacks.

I. Introduction

Taming of turbulent dynamics exhibited by spatially extended nonlinear systems is of much practical interest. The growing interest in this field stems from the pioneering work done by Ott et al.[1] in controlling chaos. Since then chaos has been controlled in various experimental systems[2−7] using different control strategies.[8−10] These efforts have been naturally extended to try to tame the complex dynamical behavior observed in distributed dynamical systems.[11−15] This control of spatiotemporal chaos leading up to the control of turbulence is an extremely complicated problem due to the existence of numerous unstable spatial modes but is immensely important too because of its possible applications in plasma, laser devices, and chemical and biological systems where both spatial and temporal dependences need to be considered. In this article we propose using feedbacks and forcing to suppress the turbulent behavior observed in a model (one spatial dimension) used for description of CO oxidation on a Pt(110) single crystal surface under UHV conditions.[16,17] This model is briefly described in the following section. In section III two different control strategies are implemented to stabilize the fixed point of the single two-dimensional oscillator. Results from implementation of different feedback and forcing techniques to an extended system composed of diffusively coupled oscillators (section II) are presented in section IV. A brief summary of results is presented in section V.

II. Numerical Model for CO Oxidation

To demonstrate suppression of spatiotemporal chaos, we choose the following model used for the description of CO oxidation on a Pt(110) single crystal surface under UHV conditions[16,17]

$$\partial_t u = -\frac{u(u-1)}{\epsilon}\left(u - \frac{v+b}{a}\right) + D\nabla^2 u \qquad (1)$$

$$\partial_t v = f(u) - v \qquad (2)$$

where the activator variable u corresponds to the coverage of the adsorbed CO, while the inhibitor variable v describes a structural change. The function $f(u)$ is of the form

$$u < 1/3 \rightarrow f(u) = 0;\ 1/3 \le u \le 1 \rightarrow f(u) = 1 - 6.75u(u-1)^2;\ u > 1 \rightarrow f(u) = 1$$

Under appropriate parameter values in one spatial dimension, the model systems exhibits traveling pulse behavior, amplitude turbulence, and phase turbulence. The system size was chosen to be 100 (dimensionless units) and was divided into 200 grid elements for simulation of the model using explicit integration algorithm with constant time and space steps (100/200) subjected to periodic boundary conditions.

III. Suppressing Oscillations in a Single Element Using Feedbacks

In this section we apply two feedback techniques to the limit cycle (period-1) dynamics exhibited by O.D.E. version of the P.D.E. (partial differential equations) model (eqs 1, 2). The two dimensional O.D.E. model was integrated using a fourth order Runge-Kutta algorithm with a fixed stepsize ($h = 0.1$). Using the feedback control we were able to stabilize the fixed point steady state of the model system.

A. Feedback: $\gamma(u(t) - u(t - \tau))$. Under the influence of the above mentioned control, the altered dynamics of the single oscillator are represented by

$$\dot{u} = -\frac{u(u-1)}{\epsilon}\left(u - \frac{v+b}{a}\right) + \gamma(u(t) - u(t-\tau)) \qquad (3)$$

$$\dot{v} = f(u) - v \qquad (4)$$

The function $f(u)$ is of the form

$$u < 1/3 \rightarrow f(u) = 0;\ 1/3 \le u \le 1 \rightarrow f(u) = 1 - 6.75u(u-1)^2;\ u > 1 \rightarrow f(u) = 1$$

Figure 1 shows the dynamical evolution of the system. Prior to iteration number 5000 the system exhibits period-1 oscillation and subsequently converges to the stabilized fixed point under

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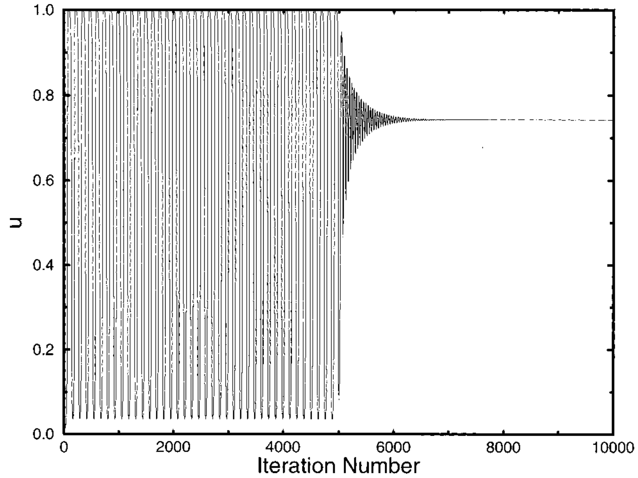

Figure 1. Dynamical evolution of the single oscillator without (<5000) control and under the influence of control (>5000) of the type as discussed in section III A. The system parameters are $a = 0.84$, $\epsilon = 0.12$, $b = -0.045$ and the control parameters are $\gamma = -0.4$ and $\tau = 10$.

the influence of the control of the type as shown in eq 3. Therefore, under the influence of the superimposed feedback the previously unstable focus is converted to a stable focus (indicated by the dampening of the period-1 oscillation). Local stability analyses of eq 3 indicate that the eigenvalues of the fixed point are a function of the control constant (γ) and that the real part of the eigenvalue actually switches sign (positive $\rightarrow$ negative) for a certain minimum value of $\gamma = 0.385$. Upon successful stabilization of the steady state, the control signal goes to zero as ($u(t) = u(t - \tau)$). However, for lower values of $\gamma < 0.38$ we were able to target on a whole array of period-1 dynamics of different amplitudes. Figure 2a shows confinement of final dynamics on one such limit cycle beyond iteration number 5000. The nonvanishing control signal is plotted in Figure 2b. This is analogous to altering dynamics via a nonvanishing feedback.

B. Feedback: $\gamma(u - uf)$. For the implementation of the feedback of the type mentioned above, one requires the location of the unstable fixed point (target state; u_f). The controlled dynamics under the influence of the feedback can be written as

$$\dot{u} = -\frac{u(u-1)}{\epsilon}\left(u - \frac{v+b}{a}\right) + \gamma(u - u_f) \qquad (5)$$

$$\dot{v} = f(u) - v \qquad (6)$$

The function $f(u)$ is of the form

$$u < 1/3 \rightarrow f(u) = 0;\ 1/3 \le u \le 1 \rightarrow f(u) = 1 - 6.75u(u-1)^2;\ u > 1 \rightarrow f(u) = 1$$

Figure 3 represents the dynamical evolution of the system without control (iteration < 5000) and with the control implemented (iteration > 5000). The control signal upon successful stabilization goes to zero as the dynamics converge on to the target fixed point state (u_f). Similar to the results discussed in the previous subsection, by decreasing γ targeting of an entire array of limit cycles (different amplitudes) was achieved with a nonvanishing control signal.

IV. Suppression of Chemical Turbulence in the Extended System

In this section we consider the extended system with periodic boundary conditions studied extensively by Bär et. al.[16,17] and discussed briefly in section II. The diffusively coupled system exhibits turbulent dynamics for the following parameter values ($a = 0.84$, $\epsilon = 0.12$, $b = -0.045$, and $D = 1/5.2$.) The implemented control strategies are presented in different subsec-

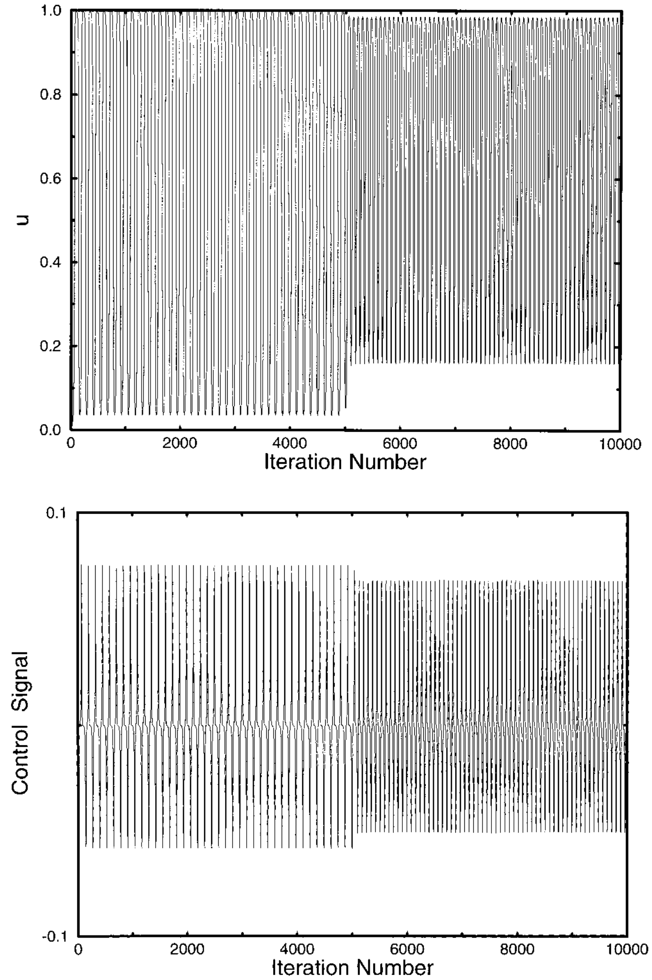

Figure 2. Successful targeting of a period-1 orbit (different amplitude and frequency) under the influence of the control of the type in section III A. The system parameters are $a = 0.84$, $\epsilon = 0.12$, $b = -0.045$ and the control parameters are $\gamma = -0.2$ and $\tau = 10$. (a) Shows the dynamical evolution of the system. Control is implemented subsequent to iteration number 5000. (b) Shows the corresponding nonvanishing control signal of the same time period.

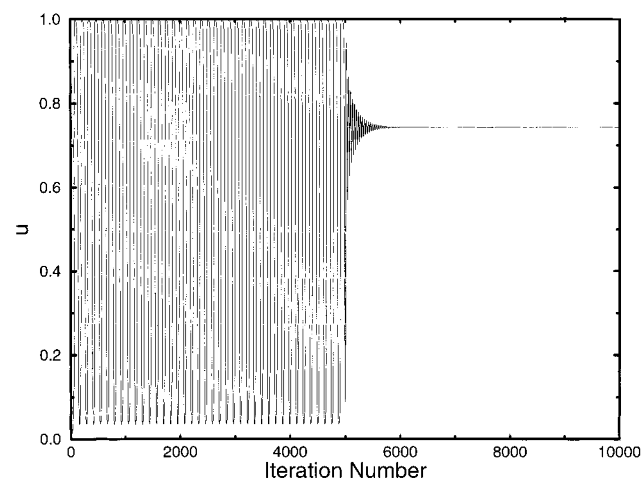

Figure 3. Dynamical evolution of the single oscillator without (<5000) control and under the influence of control (>5000) of the type as discussed in section III B. The system parameters are $a = 0.84$, $\epsilon = 0.12$, $b = -0.045$ and the control parameter is $\gamma = -0.7$.

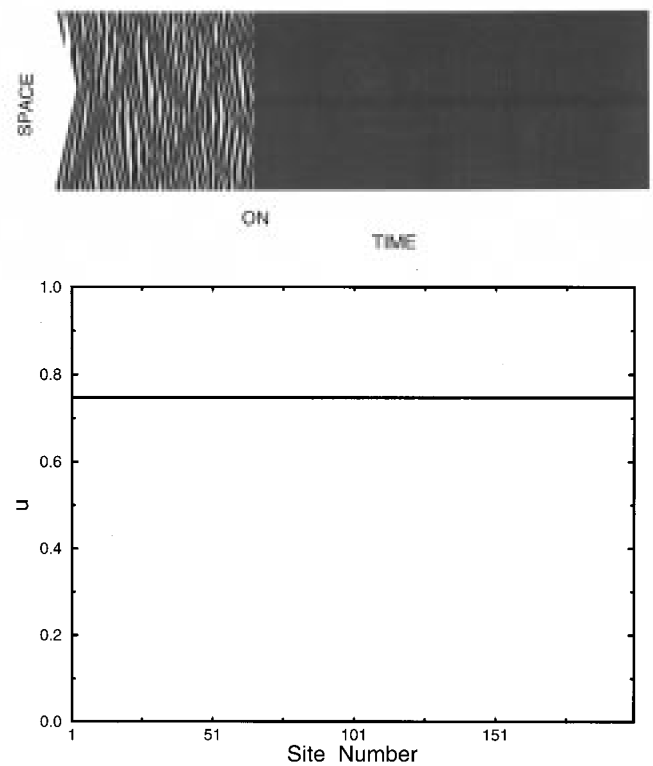

Figure 4. Control of the turbulent behavior via stabilization of the homogeneous state for 200 diffusively coupled oscillators (section II) using the control as discussed in section IV A.1. The system parameters are $a = 0.84$, $\epsilon = 0.12$, $b = -0.045$ and $D = 1/5.2$ and the control parameter are $\gamma = -0.4$ and $\tau = 10$. (a) Space−time portrait prior and subsequent to (indicated by "ON") implementation of the control signal. Every 20th step is plotted along the time axis. (b) Depicts the space amplitude profile of the controlled dynamics. It clearly indicates that the target homogeneous state is the previously unstable fixed point solution of the single oscillator.

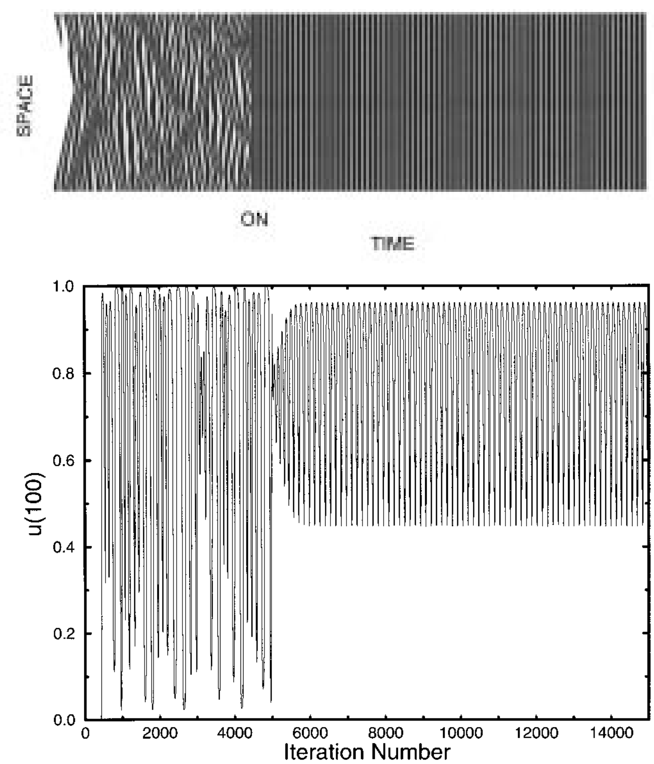

Figure 5. Suppression of chemical turbulence via stabilization of the periodic (spatially homogeneous yet temporally periodic) solution for 200 diffusively coupled oscillators (section II) using the control as discussed in section IV A.1. The system parameters are $a = 0.84$, $\epsilon = 0.12$, $b = -0.045$ and $D = 1/5.2$ and the control parameters are $\gamma = -0.1$ and $\tau = 4$. (a) Space−time portrait before and subsequent to (indicated by "ON") implementation of the control signal. Every 20th step is plotted along the time axis. (b) The local time series of the 100th cell prior and subsequent to the implementation of the control. The effect of the control is analogous to that observed in Figure 2a.

tions depending on whether or not the local state of the system is required for their successful implementation. All of the discussed feedback and forcing strategies are able to suppress the turbulent dynamics via stabilization of fixed point state and/or by stabilizing periodic (spatially homogeneous and temporally periodic) solutions. The robustness of all the results presented in this section were checked by adding a small yet finite amount of random fluctuations.

A. Local Feedback Techniques. All of the feedbacks considered in this subsection require information of the local state for successful suppression of the turbulent dynamics. Although the feedback is implemented locally, the feedback superimposed to the evolution equation is the difference between the local state and a global observable (local/global composite).

1. Feedback: $\gamma(u_i(t) - 1/N\sum_{i=1}^{N}u_i(t - \tau))$. The altered dynamics under the influence of the above feedback control are represented by

$$\partial_t u = -\frac{-1u(u-1)}{\epsilon}\left(u - \frac{v+b}{a}\right) + D\nabla^2 u + \gamma\left(u_i(t) - \frac{1}{N}\sum_{i=1}^{N}u_i(t-\tau)\right) \quad (7)$$

$$\partial_t v = f(u) - v \quad (8)$$

The functional form of $f(u)$ remains the same as used earlier. Using the control of the type in eq 7 we were able to stabilize both the homogeneous and/or the temporally periodic state depending on the value of γ and τ. Figure 4a shows the space−time plot where the control is initiated at the 5000th iteration and the dynamics stabilize on the homogeneous state. Figure 4b shows the space−amplitude plot for the controlled system. It clearly exhibits the stabilization of the target fixed point (homogeneous) state ($u_i = u_f = 0.7432$). Also, the control signal vanishes upon successful stabilization. Figure 5a shows the space−time plot for the control at a lower value of γ. In this case the control stabilizes a temporally periodic state. The local time series of the 100th cell (Figure 5b) shows a period-1 oscillation (smaller amplitude) subsequent to the application of the control (similar to Figure 2a). Also, (similar to Figure 2b) the superimposed feedback signal is nonvanishing.

2. Feedback: $\gamma(u_i - u_F)$. The controlled dynamics under the influence of this feedback are represented by

$$\partial_t u = -\frac{-1u(u-1)}{\epsilon}\left(u - \frac{v+b}{a}\right) + D\nabla^2 u + \gamma(u_i - u_F) \quad (9)$$

$$\partial_t v = f(u) - v \quad (10)$$

The control of eq 9 was also able to stabilize the homogeneous state (Figure 6). Moreover, similar to the space−time plot of Figure 5a, suppression of turbulent dynamics was also achieved via stabilization of a spatially homogeneous and temporally periodic state with a nonvanishing control signal.

B. Global Feedback Techniques. The obvious advantage of using global feedback techniques is the enhanced relevance

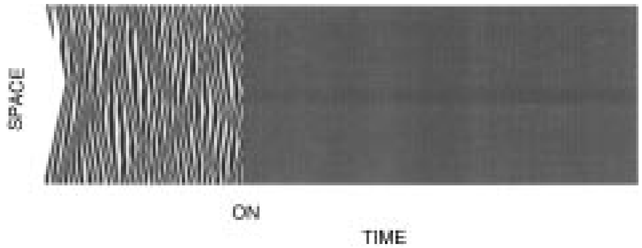

Figure 6. Space−time portrait for the coupled oscillator system with the implementation of the feedback control (indicated by "ON") as discussed in section IV A.2. The system parameters are $a = 0.84$, $\epsilon = 0.12$, $b = -0.045$ and $D = 1/5.2$ and the control parameter is $\gamma = -0.93$. Every 20th step is plotted along the time axis.

to experimental situations. The feedbacks considered in this section involve superimposing a global observable or a difference of two global observables to the dynamical equation.

1. Feedback: $\gamma(1/N\sum_{i=1}^{N} u_i(t) - 1/N\sum_{i=1}^{N} u_i(t-\tau))$. This control involves computing the difference of global averages at two different times and feeding it back into the system. The system under the influence of the control is represented by

$$\partial_t u = -\frac{-1u(u-1)}{\epsilon}\left(u - \frac{v+b}{a}\right) + D\nabla^2 u - \gamma\left(\frac{1}{N}\sum_{i=1}^{N} u_i(t) - \frac{1}{N}\sum_{i=1}^{N} u_i(t-\tau)\right) \quad (11)$$

$$\partial_t v = f(u) - v \quad (12)$$

A control of this type is plausible in an actual experimental system as the superimposed feedback can be acquired online from the experiments. Using this control we were able to stabilize spatially homogeneous states which are periodic in time. Figure 7a shows the space−time plot for one such control exhibiting stabilization on a temporally periodic state. The space−amplitude plot (Figure 7b) illustrates the point that the final state is spatially homogeneous and temporally periodic.

2. Feedback: $\gamma 1/N\sum_{i=1}^{N} u_i(t-\tau)$. Global delayed feedback has been used to control turbulence in the complex Ginzburg−Landau equation.[14] In this subsection we implement a global feedback with delay to the model system exhibiting chemical turbulence. The dynamics under the influence of the control are represented by

$$\partial_t u = -\frac{-1u(u-1)}{\epsilon}\left(u - \frac{v+b}{a}\right) + D\nabla^2 u - \gamma\frac{1}{N}\sum_{i=1}^{N} u_i(t-\tau) \quad (13)$$

$$\partial_t v = f(u) - v \quad (14)$$

Figure 8 shows the results of implementation of the global feedback control. Control is attained on a homogeneous state; however, the control signal remains a nonvanishing entity. By varying the value of γ we were able to stabilize a wide array of nonturbulent oscillatory states. It should be possible to apply this global feedback control to actual systems.

C. Forcing: γ * sin(wt). In this case one of the sites (#1) of the diffusively coupled oscillators is perturbed with a periodic forcing of the form above. Under the influence of forcing, the altered dynamics of one of the oscillators (i=1) (the evolution

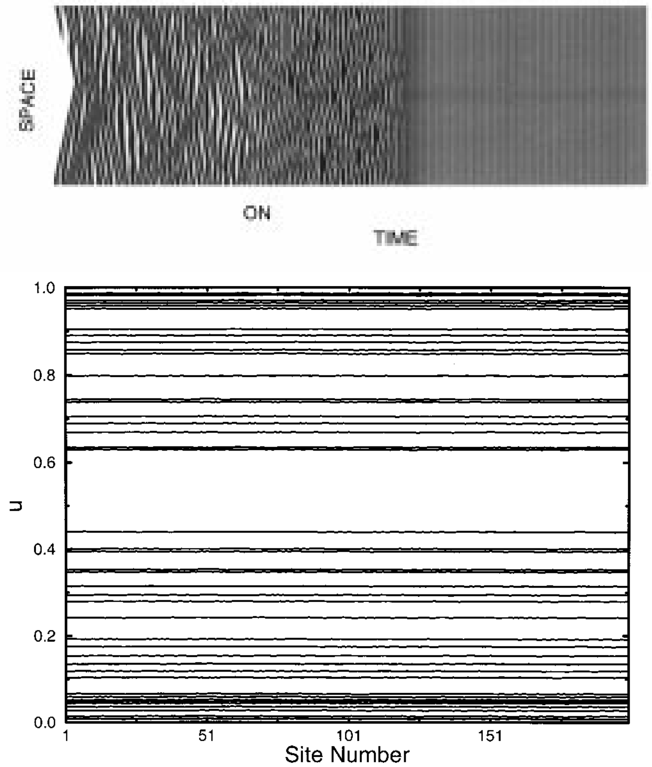

Figure 7. Control of the turbulent behavior via stabilization of the temporally periodic state for 200 diffusively coupled oscillators (section II) using the control as discussed in section IV B.1. The system parameters are $a = 0.84$, $\epsilon = 0.12$, $b = -0.045$ and $D = 1/5.2$ and the control parameters are $\gamma = -0.9$ and $\tau = 10$. (a) Space time portrait prior and subsequent to (indicated by "ON") implementation of the control signal. Every 20th step is plotted along the time axis. (b) The space−amplitude profile of the controlled dynamics. It illustrates the point that the target state is spatially homogeneous and temporally periodic.

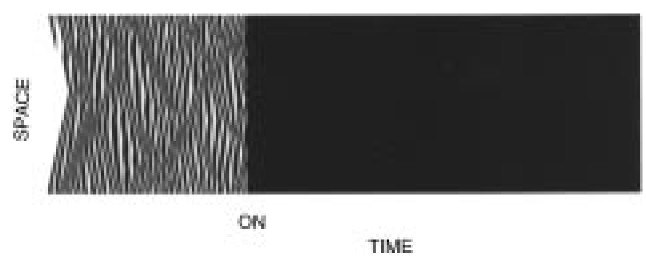

Figure 8. Space−time portrait for the coupled oscillator system with the implementation of the global feedback control (indicated by "ON") as discussed in section IV B.2. The control results in stabilization of the homogeneous state with a nonvanishing control signal. The system parameters are $a = 0.84$, $\epsilon = 0.12$, $b = -0.045$ and $D = 1/5.2$ and the control parameters are $\gamma = 0.005$ and $\tau = 8$. Every 20th step is plotted along the time axis.

equations for the remaining oscillators is unchanged) is represented by

$$\partial_t u = -\frac{-1u(u-1)}{\epsilon}\left(u - \frac{v+b}{a}\right) + D\nabla^2 u - \gamma * \sin(wt) \quad (15)$$

$$\partial_t v = f(u) - v \quad (16)$$

Figure 9 shows the space−time plot for the extended system under the effect of local forcing. For the appropriate choice of w and γ it clearly exhibits the induction of order and its subsequent propagation up until complete suppression of

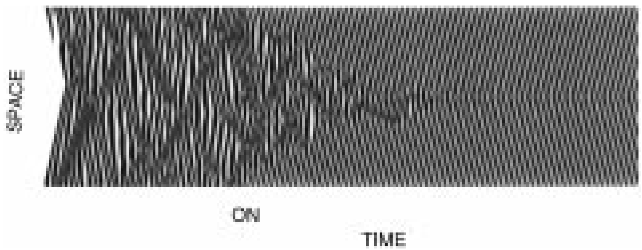

Figure 9. Space time portrait for the coupled oscillator system with the implementation of forcing to a single site (site #1; indicated by "ON") as discussed in section IV C. These local periodic perturbations propagate to the neighboring cells, resulting in the emergence of global order. The system parameters are $a = 0.84$, $\epsilon = 0.12$, $b = -0.045$ and $D = 1/5.2$ and the control parameters are $\gamma = -0.5$ and $w = 0.05$. Every 20th step is plotted along the time axis.

turbulent dynamics is achieved. The stabilized state is a stable traveling pulse train propagating in one spatial dimension. This control via local periodic perturbations is applicable in experimental situations and is successful as long as the external forcing is "ON".

V. Conclusions

In this article we have demonstrated successful suppression of turbulent behavior observed in a numerical model for a spatially extended reaction−diffusion system. The stabilized system corresponded to the homogeneous state and/or temporally periodic state depending on the type of feedback used and/or the value of γ chosen. The feedback control described in section IV (section IV A.1 − IV A.2) requires information about the local state of the system for successful implementation; however, it does not necessary imply that they are not applicable to experiments. The controlling feedback in both of these strategies is proportional to the difference between the local state and a global variable. Therefore, the correctional feedback superimposed onto a site is related to the local dynamics at that site. It is possible to envisage a setup where the response on different sites of an extended system is a function of the local state of individual sites. The following two global control strategies (IV B.1, B.2) rely on feedbacks involving global observables and in general are more relevant to experimental situations. Finally, results for external (periodic) forcing indicate that generation of global order is achieved even though the perturbations are applied locally to a single site. The final dynamics in this case is a propagating pulse train traversing the system.

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